Renormalization Group Equations

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RGEs in SUSY phenomenology

- In MSSM gauge couplings spectacularly unify at point within experimental errors in their values.
- Knowing initial values of some parameters (i.e. M_Z scale) we let parameters evolve with respect to RGEs.
- RGEs running gives us gauge unification scale that can be used as starting point M_X where all scalars and all gauginos have common mass and all prefactors of the trilinear soft scalar terms are the same.
- We use model input (i.e. mSUGRA models where we set A_0, m_0 and M_0) for initial values and let parameters evolve from $M_X = 10^{16} GeV$ to $100 GeV$ to obtain constraints to parameter space.
- Parameters in model are free or constrained by experiment and/or relations among themselves - since evolution of parameters is coupled via RGEs, choice for free parameters affects constrained.
- RGEs play crucial role in (SUSY) phenomenology - theoretical study is no longer restricted to specific scale.

Derivation of RGEs

- RPT: we start by separating bare Lagrangian to renormalized part and part including counterterms fixed by definition of physical parameters.
- For RG study we only need relations between bare and renormalized parameters of theory (example for g_s in SM)

$$\begin{aligned} g_{sB}\mu^{-\epsilon} &= (Z_3^G)^{-3/2} Z_1^{3G} g_s = (Z_3^G)^{-1} (Z_1^{4G})^{1/2} g_s \\ &= (Z_2^{c_G})^{-1} (Z_3^G)^{-1/2} Z_1^{c_G} g_s = (Z_{2i}^{q_L})^{-1} (Z_3^G)^{-1/2} Z_{1i}^{q_L G} g_s \\ &= (Z_{2i}^{u_R})^{-1} (Z_3^G)^{-1/2} Z_{1i}^{u_R G} g_s = (Z_{2i}^{d_R})^{-1} (Z_3^G)^{-1/2} Z_{1i}^{d_R G} g_s . \end{aligned}$$

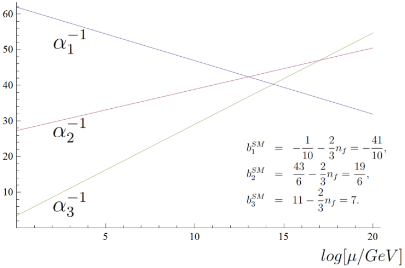
- After finding renormalization constants Z we derive beta functions.

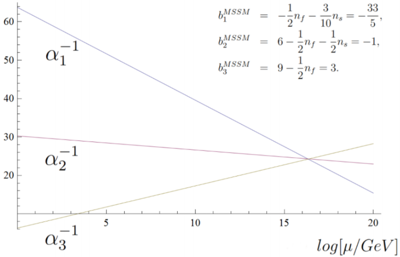
$$g_{i,B}\mu^{-\epsilon} = g_i + a_i^{(1)} \frac{g_i^3}{(4\pi)^2} \frac{1}{\epsilon} + \ldots \rightarrow \beta_i = 2 a_i^{(1)} \frac{g_i^3}{(4\pi)^2} .$$

Derivation of RGEs

- So far, we have completely derived one-loop RGEs for SM and MSSM+3N using *ab initio* calculation, starting with Lagrangian in interaction basis, in MS scheme via dimensional regularization.

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(m_Z) + \frac{b_i}{2\pi} ln \frac{\mu}{m_Z}$$





One-loop RGEs for MSSM+3N

- coupling constants

$$\beta_{g'} = 11\frac{g'^3}{(4\pi)^2}, \quad \beta_g = \frac{g^3}{(4\pi)^2}, \quad \beta_{g_s} = -3\frac{g_s^3}{(4\pi)^2}.$$

- Yukawa couplings

$$\beta_{Y_e} = \frac{1}{(4\pi)^2}\left((-3(g'^2+g^2) + Tr(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d))\mathbb{1} + 3Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu\right)Y_e,$$

$$\beta_{Y_\nu} = \frac{1}{(4\pi)^2}\left(\frac{1}{2}(g'^2-3g^2) + \frac{1}{2}Tr(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u))\mathbb{1} + \frac{1}{2}Y_e^\dagger Y_e + \frac{3}{2}Y_\nu^\dagger Y_\nu\right)Y_\nu,$$

$$\beta_{Y_d} = \frac{1}{(4\pi)^2}\left((-\frac{7}{9}g'^2 - 3g^2 - \frac{16}{3}g_s^2 + Tr(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d))\mathbb{1} + Y_e^\dagger Y_e + 3Y_d^\dagger Y_d\right)Y_d,$$

$$\beta_{Y_u} = \frac{1}{(4\pi)^2}\left((-\frac{13}{9}g'^2 + 3g^2 + \frac{16}{3}g_s^2 + Tr(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u))\mathbb{1} + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d\right)Y_d.$$

- running VEVs (in Feynman gauge)

$$\gamma(h_1) = \frac{1}{(4\pi)^2}\left(-Y_e^\dagger Y_e - 3Y_d^\dagger Y_d\right),$$

$$\gamma(h_2) = \frac{1}{(4\pi)^2}\left(-Y_\nu^\dagger Y_\nu - 3Y_u^\dagger Y_u\right).$$

One-loop RGEs for MSSM+3N

- SUSY breaking terms I: scalar masses

$$\beta_{h_1} = \frac{1}{8\pi^2}\Big(m^2_{h_1} Tr(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d) + m^2_{\tilde{\ell}_L} Y_e^\dagger Y_e + m^2_{\tilde{e}_R} Y_e Y_e^\dagger + 3m^2_{\tilde{q}_L} Y_d^\dagger Y_d + 3m^2_{\tilde{d}_R} Y_d Y_d^\dagger +$$

$$+ Tr(A_e^\dagger A_e + 3A_d^\dagger A_d) - g'^2 M_1^2 - 3g^2 M_2^2 - \frac{g'^2}{2} Tr\{Ym\}\Big),$$

$$\beta_{h_2} = \frac{1}{8\pi^2}\Big(m^2_{h_2} Tr(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u) + m^2_{\tilde{\ell}_L} Y_\nu^\dagger Y_\nu + 3m^2_{\tilde{q}_L} Y_u^\dagger Y_u + 3m^2_{\tilde{u}_R} Y_u Y_u^\dagger +$$

$$+ Tr(A_\nu^\dagger A_\nu + 3A_u^\dagger A_u) - g'^2 M_1^2 - 3g^2 M_2^2 - \frac{g'^2}{2} Tr\{Ym\}\Big),$$

$$\beta_{\tilde{\ell}_L} = \frac{1}{8\pi^2}\Big(\frac{1}{2} m^2_{\tilde{\ell}_L} Y_e^\dagger Y_e + \frac{1}{2} Y_e^\dagger Y_e m^2_{\tilde{\ell}_L} + m^2_{h_1} Y_e^\dagger Y_e + m^2_{h_2} Y_\nu^\dagger Y_\nu + Y_e^\dagger m^2_{\tilde{e}_R} Y_e + Y_\nu^\dagger m^2_{\tilde{\nu}_R} Y_\nu +$$

$$+ A_\nu^\dagger A_\nu + A_e^\dagger A_e - g'^2 M_1^2 - 3g^2 M_2^2 - \frac{g'^2}{2} Tr\{Ym\}\Big),$$

$$\beta_{\tilde{e}_R} = \frac{1}{8\pi^2}\Big((m^2_{\tilde{e}_R} + 2m^2_{h_1}) Y_e Y_e^\dagger + Y_e Y_e^\dagger m^2_{\tilde{e}_R} + 2Y_e m^2_{\tilde{\ell}_L} Y_e^\dagger + 2A_e A_e^\dagger - 4g'^2 M_1^2 + g'^2 Tr\{Ym\}\Big),$$

$$\beta_{\tilde{\nu}_R} = \frac{1}{8\pi^2}\Big(m^2_{\tilde{\nu}_R} Y_\nu Y_\nu^\dagger + Y_\nu Y_\nu^\dagger m^2_{\tilde{\nu}_R} + 2m^2_{h_2} Y_\nu Y_\nu^\dagger + 2Y_\nu m^2_{\tilde{\ell}_L} Y_\nu^\dagger + 2A_\nu A_\nu^\dagger + 2M^\dagger Y_\nu Y_\nu^\dagger M\Big).$$

One-loop RGEs for MSSM+3N

- SUSY breaking terms I: scalar masses

$$\beta_{\tilde{q}_L} = \frac{1}{8\pi^2}\Big(\frac{1}{2}m^2_{\tilde{q}_L}(Y_d^\dagger Y_d + Y_u^\dagger Y_u) + \frac{1}{2}(Y_d^\dagger Y_d + (Y_u^\dagger Y_u)m^2_{\tilde{q}_L} + m^2_{h_1}Y_d^\dagger Y_d + m^2_{h_2}Y_u^\dagger Y_u +$$

$$+ Y_d^\dagger m^2_{\tilde{d}_R} Y_d + Y_u^\dagger m^2_{\tilde{u}_R} Y_u + A_d^\dagger A_d + A_u^\dagger A_u - \frac{1}{9}g'^2 M_1^2 - 3g^2 M_2^2 - \frac{16}{3}g_s^2 M_3^2 + \frac{g'^2}{6}Tr\{Ym\}\Big),$$

$$\beta_{\tilde{u}_R} = \frac{1}{8\pi^2}\Big(m^2_{\tilde{u}_R} Y_u Y_u^\dagger + Y_u Y_u^\dagger m^2_{\tilde{u}_R} + 2m^2_{h_2} Y_u Y_u^\dagger + 2Y_u m^2_{\tilde{q}_L} Y_u^\dagger +$$

$$+ 2A_u A_u^\dagger - \frac{16}{9}g'^2 M_1^2 - \frac{16}{3}g_s^2 M_3^2 - \frac{2}{3}g'^2 Tr\{Ym\}\Big),$$

$$\beta_{\tilde{d}_R} = \frac{1}{8\pi^2}\Big(m^2_{\tilde{d}_R} Y_d Y_d^\dagger + Y_d Y_d^\dagger m^2_{\tilde{d}_R} + 2m^2_{h_1} Y_d Y_d^\dagger + 2Y_d m^2_{\tilde{q}_L} Y_d^\dagger +$$

$$+ 2A_d A_d^\dagger - \frac{4}{9}g'^2 M_1^2 - \frac{16}{3}g_s^2 M_3^2 + \frac{1}{3}g'^2 Tr\{Ym\}\Big),$$

where

$$Tr\{Ym\} = \sum_{n_g}(m^2_{\tilde{q}_L} - 2m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - m^2_{\tilde{\ell}_L} + m^2_{\tilde{e}_R}) - m^2_{h_1} + m^2_{h_2}.$$

One-loop RGEs for MSSM+3N

- SUSY breaking terms II: trilinear scalar terms

$$\beta_{A_e} = \frac{1}{16\pi^2}\Big(A_e(-3g'^2 - 3g^2 + Tr(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d)) + Y_e(6g'^2 M_1^2 + 6g^2 M_2^2 + Tr(2Y_e^\dagger A_e + 2Y_d^\dagger A_d)) +$$
$$+ 4Y_e Y_e^\dagger A_e + 5A_e Y_e^\dagger Y_e + 2Y_e Y_\nu^\dagger A_\nu + A_e Y_\nu^\dagger Y_\nu\Big),$$

$$\beta_{A_\nu} = \frac{1}{16\pi^2}\Big(A_\nu(-g'^2 - 3g^2 + Tr(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u)) + Y_\nu(6g'^2 M_1^2 + 6g^2 M_2^2 + Tr(2Y_\nu^\dagger A_\nu + 6Y_u^\dagger A_u)) +$$
$$+ 4Y_\nu Y_\nu^\dagger A_\nu + 5A_\nu Y_\nu^\dagger Y_\nu + A_\nu Y_e^\dagger Y_e + 2Y_\nu Y_e^\dagger A_e\Big),$$

$$\beta_{A_d} = \frac{1}{16\pi^2}\Big(A_d(-\frac{7}{9}g'^2 + 3g^2 - \frac{16}{3}g_s^2 + Tr(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d)) + Y_d(\frac{14}{9}g'^2 M_1^2 + 6g^2 M_2^2 + \frac{32}{3}g_s^2 M_3^2 +$$
$$+ Tr(6Y_d^\dagger A_d + 2Y_e^\dagger A_e)) + 4Y_d Y_d^\dagger A_d + 5A_d Y_d^\dagger Y_d + A_d Y_u^\dagger Y_u + 2Y_d Y_\nu^\dagger A_\nu\Big).$$

$$\beta_{A_u} = \frac{1}{16\pi^2}\Big(A_u(-\frac{13}{3}g'^2 - 3g^2 - \frac{16}{3}g_s^2 + Tr(Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u)) + Y_u(\frac{26}{9}g'^2 M_1^2 + 6g^2 M_2^2 + \frac{32}{3}g_s^2 M_3^2 +$$
$$+ Tr(6Y_u^\dagger A_u + 2Y_\nu^\dagger A_\nu)) + 4Y_u Y_u^\dagger A_u + 5A_u Y_u^\dagger Y_u + A_u Y_d^\dagger Y_d + 2Y_u Y_d^\dagger A_d\Big)$$

- Superpotential mixing mass and bilinear soft scalar coupling

$$\beta_\mu = \frac{1}{16\pi^2}\mu\Big(-g'^2 - 3g^2 + Tr(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3(Y_d^\dagger Y_d + Y_u^\dagger Y_u))\Big),$$
$$\beta_B = \frac{1}{8\pi^2}\Big(g'^2 M_1 + 3g^2 M_2 + Tr(Y_e^\dagger A_e + Y_\nu^\dagger A_\nu + 3(Y_d^\dagger A_d + Y_u^\dagger A_u))\Big).$$

Future plans and questions

- RGEs solutions are crucial in any calculation of observables.
- Loop computations are also needed for loop observables.
- Next step - two loop RGEs for MSSM+3N.
- Automatization of calculations in two independent ways via:
  - general purpose packages to obtain loop results needed for loop observables and RGEs,
  - SARAH to calculate RGEs and check consistency.
- One-loop calculation results can be used as check on computer tools and definition of models before two-loop exstension.

Future plans and questions

- It is well known that Dimensional Regularization breaks SUSY - need for Dimensional Reduction and scheme independent Algebraic Renormalization.
- Slavnov-Taylor Identities - their derivation for Algebraic Renormalization, connection with RPT's renormalization factors.
- Choice of gauge: i.e. when is Background Field Gauge needed or useful?
- gauge dependent calculations
- GOAL: setting theoretical framework for phenomenology calculations and automatization as little as possible model dependent.

Thank you!

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